Feds with EDS: Searching for the optimal explosive scanning system

COMAP Competition
6-10 February 2003

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Abstract

Meeting the demand for aviation security following September 11 accomplishes the dual goals of fighting terrorism and strengthening the economy by restoring air traveler confidence. In order to satisfy these demands in a cost-effective manner, we derived a model that determines the minimum number of EDS machines needed to process the peak travel hour's quantity of bags at an airport, thereby being sufficient for the rest of the day. To ensure the optimal usage of these machines, we devised a flight scheduling algorithm that allows us to regulate the flow of bags, allowing the machines to operate at full capacity as long as possible. We also determined the importance of the mandatory time at which all airlines require passengers to arrive early. Through analyzing the cost functions of EDS and ETD machines, we showed that, in the long run, EDS machines are cheaper due to their lower operating cost. New and improved technology should be pursued, but unless a new machine has a significantly lower operating cost than the EDS, it will probably continue to be cheaper to use EDS until the lifespan of the EDS machine expires.

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Introduction

A May 2002 Transportation Security Administration (TSA) press release outlines the pilot testing of different baggage screening programs at three airports. One airport used all Explosive Trace Detection (ETD) machines, one used all Explosive Detection System (EDS) machines, and a third airport used half and half. With some mathematical ingenuity, we intend to show that these pilot tests were unnecessary.

Our model, in its purest form, is rather simple, dependent mainly upon statistical data and probabilities to fill in the elements of the rate equation. We will focus on maximization of both the productivity of the machines and the amount of time they have to process the highest peak in checked bags, thus being sufficient for other times. We will show the importance of proper flight schedule planning and the ideal method for scheduling.

The implementation of the model's conclusions will save the government and airports money in purchasing and installing expensive machinery. Security will be paramount; minimizing passenger inconvenience will be the secondary concern, but under our model, we eliminate or, at least, minimize expected delays. By extending our model, we can also potentially find the optimal amount of time before takeoff at which passengers should be required to arrive at the airport. To minimize cost, this time may need to be increased or decreased, depending on experimental data. Ideally, the model ensures security, creates economic growth, relieves stress, and prevents headaches and nausea, with few or no side effects.

Far better than snake oil, let us now outline our assumptions and then describe the model.

General Assumptions

The following have been assumed unless temporarily revoked elsewhere in the paper.

We assume all data as given on the Technical Information Sheet (TIS)

We assume that, if all of our assumptions hold, it is unsatisfactory to expect flight delays due to the explosive detection machines.

However, we assume that a 15-minute delay is considered on time, according to FAA policy.[1] If, on a normal day, empirical data differing from our assumptions causes a delay of less than 15 minutes, we assume that such delays will likely go unnoticed.

We assume that the percentage of planes that are cancelled *before* baggage is checked is negligible.

We assume no extreme unforeseen circumstances, e.g. striking workers, that might affect baggage screening and flight departures.

We assume there will be no major disturbances in the *Force* during the peak hour.

We assume that the number of passengers who will check more than two bags is negligible.

We also assume that by the time our plan is fully implemented, all airports will have EDS or other scanning machines functional; thus, we will not need to rescan the bags belonging to connecting passengers originating elsewhere.

The model requires that a system of bag queuing and prioritizing process will be in place. For example, if one person arrives 2 hours ahead of their scheduled flight, and another person arrives at the same time, but their flight is in 45 minutes, then the latter person's checked bags will be processed by an EDS before the former's. We assume this system to exist and, furthermore, that it is more cost-effective to pay someone to prioritize the bags than to buy unnecessary EDS machines to compensate for chaos's inefficiencies. How to implement such a system exceeds the scope of this paper.

We assume that prioritizing negates the benefits of passengers arriving earlier than mandatory time.

We assume that the answer to life, universe, and everything is 42.[2]

For our initial EDS model, we assume no significant delay in having to re-scan or hand-examine bags due to false positives; thus, we ignore it in our calculations.

We assume that the throughput rate of bags per hour per EDS machine can be increased, subject to the TIS-specified limit of 210 bags per hour per EDS machine, by educating the operators of the machines.

We ignore the cost of repurchasing EDS or ETD machines due to defects and breakdowns. We also assume that performing scheduled maintenance on these machines will reduce the chance of machine failure. Therefore, we include the cost of maintenance in annual operational costs.

We ignore potential lines at the airline check-in desk.

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[1] Mead, Kenneth M., "Challenges Facing TSA"

[2] For a proof, see Hitchhiker's Guide to the Galaxy, by Douglas Adams

1. Task 1

1.1 The Model

$$Q_{EDS} = \left\lceil \frac{\phi \sum_{i=1}^{8} (t_i n_i P_{seatsfilled_i})}{\Omega \ell (1 + \tau - \mu)} \right\rceil \qquad \textit{(eq. 1)}$$

Q_{EDS} = quantity of EDSs needed
ℓ = throughput rate of each machine, in terms of bags/hour/machine
τ = minimum early passenger arrival time, in hours, i.e. how long before departure the airline closes bag check-in
μ = travel time of one bag between EDS and the plane, in hours
t_i = number of seats on flight of type *i* (t_1 = 34, t_2 = 46, etc.)
n_i = number of flights of type *i* during the peak hour
$P_{seatsfilled_i}$ = estimated percentage of seats filled in flights of type *i*
ϕ = summation shift constant, defined below
Ω = percent of time that the EDS is operational (given as 92%)

1.1.1 Deriving the model

We could stop here, as we believe that we have shown Q_{EDS}, q.e.d. However, so as to humor the reader, we shall elaborate. To begin, we establish that we are dealing with a model of rates, such that B_{peak}, the number of bags in the peak hour, equals the rate of bags processed multiplied by the amount of time *T*. The rate of bags per hour depends on ℓ, the number of bags that one machine can process in one hour, times Q_{EDS}, the number of EDSs. Combining these relations, we create a model that incorporates the necessary variables, solved for Q_{EDS} (our variable of interest):

$$Q_{EDS} = \frac{B_{peak}}{\ell T}$$

We should still verify that the units combine correctly,

$$machines = \frac{bags}{\frac{bags}{machine \cdot hours} \cdot hours}.$$

As it is given that each EDS is operational only a percentage of the time, we must discount the time by this constant, Ω, yielding

$$Q_{EDS} = \left\lceil \frac{B_{peak}}{\Omega \ell T} \right\rceil.$$

We add the ceiling brackets because the number of EDS must be whole; we postpone the justification for always taking the ceiling value until after we have calculated some values. We will now show the derivations of B_{peak} and *T*. Discussion of the latter variable will be deferred for the moment.

1.1.2 Deriving B_{peak}

We define B_{peak} as the aggregate of the number of bags on each flight. To calculate the number of bags on one flight, we must multiply the number of passengers on the flight by the number of bags each carries. Both values depend on probabilities. The average number of bags per passenger, $\bar{b}$, is $b_1 + 2b_2$, with b_1 and b_2 defined as the percentage chance that a passenger will check one bag and two bags, respectively. Likewise, the data table found in the Technical Information Sheet (TIS)[3] lists seating capacities of eight flight types involved in our peak hour, but the number of passengers per flight depends on the probability that those seats are filled, $P_{seatsfilled_i}$. By multiplying the number of bags on one flight, $\bar{b}\,t_i P_{seatsfilled_i}$, by the number of flights of the same type departing in the peak hour, n_i, we get the total number of bags on all flights of type *i*. By summing up all eight flight types, we arrive at B_{peak}:

$$B_{peak} = \sum_{i=1}^{8} [t_i (P_{seatsfilled_i})(\bar{b})(n_i)] \qquad \textit{(eq. 2)}$$

However, a couple other factors that influence B_{peak} need consideration.

- *Flight cancellations*

  It is given in the TIS that 2% of flights are cancelled daily. However, in our flying experiences, a flight is generally not cancelled until after the bags have been checked and the passengers are waiting at the gate, or perhaps already on the flight. When forced to, airlines tend to delay flights as long as possible, canceling only after all other options have been exhausted. Thus, we will assume that the cancellation of flights does not affect the number of checked bags that the EDSs will be expected to scan for explosives.

- *Connecting passengers*

  Since all airports must scan 100% of bags being checked in, and since the typical installation location of the EDS machines are in the passenger check-in area, we assume that bags belonging to connecting passengers do not need to be rescanned, as is current FAA policy. We define the percentage of non-connecting passengers, i.e. those originating in our airport, as $P_{orig} : 0 \le P_{orig} \le 1$.

Including these factors into equation (2), we get:

$$B_{peak} = \sum_{i=1}^{8} [t_i (P_{orig})(P_{seatsfilled_i})(\bar{b})(n_i)]$$

By collecting the constants $\bar{b}$ and P_{orig}, factoring them out of the summation quantity, and defining our summation shift constant as $\phi = \bar{b} P_{orig}$, we arrive at

$$B_{peak} = \phi \sum_{i=1}^{8} (t_i n_i P_{seatsfilled_i})$$

Substituting this into $Q_{EDS} = \dfrac{B_{peak}}{\Omega \ell T}$, we get the following:

$$Q_{EDS} = \frac{\phi \sum_{i=1}^{8} (t_i n_i P_{seatsfilled_i})}{\Omega \ell T}$$

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[3] See Appendix A

1.1.3 The Cost Function Caveat: "It's the economy, stupid."

The ultimate goal of this assignment is to minimize cost. This model's cost function (in thousands of dollars) for Airport A is $C_A(Q_{EDS}) = Q_{EDS}(1100 + \omega)$, and for Airport B, $C_B(Q_{EDS}) = Q_{EDS}(1080 + \omega)$, where ω is the operating cost per machine, and 1100 and 1080 are the costs to purchase and install the machines at each airport, given TIS data. The latter two quantities are, for the purposes of this model, immutable. Thus, cost is directly dependent on Q_{EDS}, and for the current discussion, we will ignore the cost function and focus on minimizing Q_{EDS}, thereby minimizing cost.

In order to minimize Q_{EDS}, we can either reduce B_{peak} or increase ℓ or T.

Minimizing B_{peak}

Decreasing B_{peak} would involve many factors: having passengers check fewer bags or reducing the number of passengers flying during peak hour via flight cancellation or rescheduling to non-peak times. Flight cancellation – i.e. lower airline revenue, fewer choices of flights for consumers, and the semblance of ineptitude on the part of the Transportation Security Administration (TSA) and the individual airports' administrations – is clearly undesirable. Rescheduling to non-peak times seemingly would be desirable, but surely the airlines and airports have already tackled this issue in the past, as flight delays already are a chronic disorder, so further progress in rescheduling cannot be expected. Finally, requiring passengers to check fewer bags – which the threat of longer wait times might indirectly accomplish – would be unpopular among passengers; furthermore, merely suggesting passengers to bring less checked luggage cannot be relied upon. So, to minimize Q_{EDS}, we must look instead to maximize ℓ and T.

Maximizing ℓ

The problem states that ℓ, the number of bags per hour that each machine can process, is between 160 and 210. We assume that this range is dependent on the competence of the EDSs operator. Thus, by instituting a more comprehensive and extensive training regimen, we can hope to increase ℓ, thereby lowering the number of EDS devices needed. We also assume that the savings due to needing fewer machines outweigh the costs of increased training. Acknowledging that other factors could limit the machine's output, and fearing an overly optimistic view of education's benefits (big joke setup…), we have estimated ℓ to be a modest 190 bags/hour/machine.

Maximizing T

Before we begin, we should establish the schedule and procedures of an airport and some intuitive conclusions that derive from them. All airlines and airports suggest an amount of time that passengers should arrive early to ensure smooth check-in and boarding. They also have a certain time that is absolute and inflexible – not merely recommended – such that after this time, a passenger may not check-in and board his or her flight. We label this quantity of time before departure as τ and, accounting for current airline trends and data supplied by the TIS, give it the value of 45 minutes, or 0.75 hours, unless otherwise stated in the paper. Because τ is the time before a flight *at which all passengers who will board the flight must be present*, τ is also the time before a flight at which all bags will be present. We, for now, will disregard the effect of passengers arriving earlier than τ before their flight. Thus, the EDS operators can be guaranteed that they will have $\tau - \mu$ in which to process the bags for each flight, where μ is the time needed for bags to travel the labyrinth of conveyor belts, be placed on the baggage cart, and be loaded onto the plane. As we have no data for μ, we have arbitrarily set its value at 6 minutes, or 0.1 hours. Given these assigned values, EDS operators have *at least* 39 minutes, or .65 hours, to process the bags for each flight. Our task now is to maximize this amount of time.

Now, if we were to assume that the peak hour is the only hour in which flights leave the airports, EDS processing for the peak hour can begin 45 minutes before the first flight, and the last bag of the last flight must finish being processed 6 minutes before the end of the hour. Thus, we have *at most* 1 hour and 39 minutes, or 1.65 hours, to process all of the peak hour bags. Therefore the total time $T = 1 + \tau - \mu = 1.65$.

To best use this maximum time interval, we need a steady supply of bags coming in, which entails a steady flow of bags out, so that we can choose the minimum number of EDSs to purchase. A steady supply would allow The machines to operate at maximum output for the entire time interval, while an uneven distribution of bags arriving to be processed would require at certain times greater output and thus more machines and higher cost, and at other times less output, meaning idle, unproductive, expensive machines. Clearly, dealing with quantized flights and numbers of bags per flight, we cannot guarantee a perfectly steady influx of bags, but, as we will show in Task 3, we can come close enough to reap the benefits, and thus to assume a constant flow of bags in.

With this point sufficiently established to continue, now let us revoke the assumption that the peak hour is the only hour of flights at the airport. The bags in the hours immediately before and after peak, by definition fewer than B_{peak}, can be processed in less time than needed to process B_{peak}. When the peak hour's first bags arrive 45 minutes before the peak hour begins, we cannot yet assume that the EDSs will be available to process them because flights departing during the hour before peak will have bags that need to be processed.[4] In a similar manner, we cannot assume that the EDSs can process our peak hour's bags all the way up to last moment since the bags of the next hour's first flight will likely require more than a few minutes to process. So, we should expect possible encroachments of our 1.65-hour maximum time interval from EDS demand from the hour before peak and from the hour after peak, shouldn't we? Using some empirical data and some mathematical tricks, we can, however, assume that the quantity of baggage in other periods has no effect on the simple model that ignores other periods.[5] So, we define $T = 1 + \tau - \mu$ and arrive at our final version of the model,

$$Q_{EDS} = \left\lceil \frac{\phi \sum_{i=1}^{8} (t_i n_i P_{seatsfilled_i})}{\Omega \ell (1 + \tau - \mu)} \right\rceil$$

Now we are ready to define our constants and compute the optimal Q_{EDS} values.

1.2 Solving for the optimal Q_{EDS}

Calculating B_{peak}

$$B_{peak} = \sum_{i=1}^{8} (t_i n_i P_{seatsfilled_i} \bar{b} P_{orig}) \qquad \textit{(eq. 3)}$$

We will commence by examining each component of the above equation. It is given that 20% of passengers check no bags and that another 20% check just one bag; thus, the remainder of the passengers, or 60% of them, check two bags. So, the average number of bags per passenger is:

$$\bar{\mathrm{b}} = 0(0.2) + 1(0.2) + 2(0.6) = 1.4$$

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[4] Recall that, because of μ, the bags for flights departing at 1:20 must have left the EDS's by 1:14.

[5] For the said data and trickery, see Appendix B.

$P_{seatsfilled_i}$ is given to be between 70% and 100% for flights of type 1 through 3, between 60% and 100% for flights of type 4 through 7, and between 50% and 100% for type 8. Through analysis of airport data, we calculated average values[6] for $P_{seatsfilled_i}$ such that

$$P_{seatsfilled_i} = .8679 \quad \text{for } 1 \le i \le 3$$
$$P_{seatsfilled_i} = .8194 \quad \text{for } 4 \le i \le 7$$
$$P_{seatsfilled_i} = .7705 \quad \text{for } i = 8$$

Regarding connecting passengers, statistics show that, on average, 15% of passengers are from connecting flights. Data from airports serving as hubs show even higher percentages, and since Airports A and B are described as "two of the largest facilities in the region," we infer that their percentages of connecting passengers are likely higher than 15%. Lacking precise data for A and B, however, we have conservatively estimated 15%. So, the number of passengers that require scanning can be reduced by multiplying the total passengers by 0.85, or P_{orig}.

Including $\overline{b}$ and P_{orig} into equation (3), we get:

$$B_{peak} = \sum_{i=1}^{8} [t_i (0.85)(P_{seatsfilled_i})(1.4)(n_i)]$$

which, by factoring out the probabilistic constants, yields the final equation,

$$B_{peak} = 1.19 \sum_{i=1}^{8} (t_i n_i P_{seatsfilled_i}) .$$

Substituting in Airports A's and B's values for t_i and n_i, which are given in the Technical Information Sheet[7], and our values for $P_{seatsfilled_i}$, which are given above, we calculate the following:

$$\boxed{\begin{aligned} B_{peakA} &= 5286.16 \approx 5286 \text{ bags} \\ B_{peakB} &= 5682.57 \approx 5683 \text{ bags} \end{aligned}}$$

Now we can proceed to determine the minimum Q_{EDS} sufficient to process B_{peak} at each airport.

Calculating Q_{EDS}

$$Q_{EDS} = \frac{B_{peak}}{\Omega(\ell)(1 + \tau - \mu)}$$

To solve the above, we will move through each component of the equation. First, we have determined the value of B_{peak} for each airport with the formulas shown earlier in this section. We are given that an EDS is operational 92% of the time, which is denoted here as Ω. As noted previously, we will use $\ell = 190$ as an average value for the rate of bags per machine per hour. The value for τ is for now constant, as is μ: $\tau = 0.75$ and $\mu = 0.1$. Using these values and the respective values of B_{peak} for each airport, we arrive at the following values for Q_{EDS}:

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[6] For data, derivation, and justification of these numbers, see Appendix C.

[7] Please see Appendix A for raw data given in the Technical Information Sheet (TIS).

Airport A:

$$Q_{EDS} = \frac{5286}{(0.92)(190)(1+0.75-0.1)}$$

$$\boxed{Q_{EDS} = \lceil 18.33 \rceil = 19}$$

Airport B:

$$Q_{EDS} = \frac{5286}{(0.92)(190)(1+0.75-0.1)}$$

$$\boxed{Q_{EDS} = \lceil 19.70 \rceil = 20}$$

Justification for rounding up the Q_{EDS} and contemplating the "what-ifs"

To extend our model, we need to show how changes in some variables affect the amount of time needed to process B_{peak}. Here, we solved our Q_{EDS} equation for T:

$$T = \frac{B_{peak}}{\Omega \ell Q_{EDS}}.$$

By substituting 1.65 hours (simply derived from $1 + \tau - \mu$, with the stated values of $\tau = 0.75$ and $\mu = 0.1$), we determined the change in time caused by a change in one of the variables.

The ceiling of Q_{EDS}

Here we will justify why we took the ceiling of the non-integer values we calculated in Q_{EDS}, rather than the floor. The following tables tell us the delay in minutes (rounded up) resulting from varying levels of the number of occupied seats and varying values of possible ℓ. The three levels of occupied seats that are shown in this table represent the situations when all the flights during peak hour are at full capacity, when the flights are at the average capacity that we carefully estimated, and when the flights have the minimum number of seats filled. The minimum number of seats filled is determined by multiplying the total seat capacity by the minimum percentages set forth on the Technical Information Sheet. Likewise, the maximum and minimum values of ℓ are taken from the given values in the problem description, while $\ell = 190$ represents the value that we have used in most of this paper. Zeros in the tables below represent calculated delays of 0 minutes or less.

Table 1a - Delays for Airport A

ℓ	Maximum Seats Filled	Average Seats Filled	Minimum Seats Filled
160	39	14	0
190	17	0	0
210	6	0	0

Table 1b - Delays for Airport B

ℓ	Maximum Seats Filled	Average Seats Filled	Minimum Seats Filled
160	41	17	0
190	19	0	0
210	8	0	0

If we had taken the floors of the respective Q_{EDS}, 18 and 19, then with the average number of filled seats and $\ell = 190$, we would have ended up with an average delay of about 2 and 3.6 minutes. We believe that any delay is unacceptable in the average case. To uphold the dignity of the Transportation Security Administration, we have removed any delay in the average case by taking the ceiling of the original non-integer values for the number of machines (18.33 and 19.70, respectively). Thus we arrived at the values of 19 and 20 machines, respectively.

To generalize this a bit, when an airport discovers the number of machines it requires, and the number is a non-integer, then said airport should take the ceiling of that number (or, in other words, always round up to the next highest integer, even if the value is close to the integer lower than it). Otherwise, frequent delays can be expected.

1.3 Exploring ϕ

When calculating the total number of bags, we multiplied our summation by the constant of 1.19 to arrive at our model's Q_{EDS} data. This number comes from:

$$\phi = (bags / passenger)*(\%of - non - connecting - passengers)$$

The bags/passenger is dependent on the percentage that bring 0, 1, and 2 bags, which were given as 20%, 20%, and 60% respectively. We arrive at ϕ=1.19 with the following values:

$$\phi = ((0*0.2)+(1*0.2)+(2*0.6))*(0.85) = (1.4*0.85) = 1.19$$

As mentioned earlier, we estimated that the percent of non-connecting passengers to be 85% for our model. However, the percentage of non-connecting passengers is not likely to vary greatly on a daily basis, and therefore we do not explore the change of this number in our model. If Airports A and B have different non-connecting passenger values, it would only mean a mere simple modification in our calculations. However, we do need to explore the variance in the number of bags per passenger. During times such as holidays, it might be that the passengers are more likely to carry more bags. To account for this, we'll examine the extreme of each passenger carrying on two bags:

$$\phi = (2*1.00)*(0.85) = 1.7$$

Thus, since we have already examined the scenario when ϕ = 1.19, we shall explore the model when ϕ = 1.7. Using 19 machines for Airport A, and 20 machines for Airport B, we get the following results for the delay in minutes (once again, rounded up) with various occupancies on the airplanes and various rates, ℓ:

Table 2a - Airport A			
ℓ	**Maximum Seats Filled**	**Estimated Seats Filled**	**Minimum Seats Filled**
160	98	63	21
190	67	37	2
210	51	24	0

Table 2b - Airport B			
ℓ	**Maximum Seats Filled**	**Estimated Seats Filled**	**Minimum Seats Filled**
160	101	66	24
190	70	40	5
210	54	27	0

As one can easily tell, there is a much higher delay when every passenger is checking two bags in. Since we are in the mindset that passengers will carry more bags during busier times, we can also keep in mind that there will probably be more seats filled during this time period as well. So we will just examine the scenario if the all the seats are filled on a given flight. However, since these busiest times of the year occur so rarely, we believe it is not worth buying the extra machines to handle this overload, when they are not necessary during the majority of the year. Doing so would result in an increased number of idle machines for the rest of the year, which would not only be idle, but would take up more space in the airport's lobby and decrease the flow of passengers through the check-in process. This would increase the delay of flights, further decreasing the already diminishing returns from additional machines. A possible solution to this increase in baggage is to turn to more temporary solutions, such as the renting of other portable screening devices, or the temporary hiring of extra workers or K-9 dogs. This solution will be discussed further in Task 7, with the introduction of ETD machines.

So, in summary, referring back to Tables 1a and 1b, let us discuss three different scenarios. First, the worst case scenario: on the busiest day of the year in Airport A or B, for instance the Wednesday of Thanksgiving week, when every flight in the Peak Hour is absolutely full, if the EDS is operating at its lowest rate (ℓ=160), then there will only be about 50 minutes of delay. We believe this is acceptable, since on the Wednesday of Thanksgiving week, there are other delays anyhow, so the airplane would possibly be an hour late anyhow. However, at the maximum capability of the EDS, the delay would only be about 15 minutes on this same day. This is an acceptable delay time, according to the FAA.

The average case scenario: on an average day of the year in Airport A or B, when the flights are at their average occupancy, if the EDS is operating at its lowest rate (ℓ=160), then there will be about 25 minutes of delay. However, if the EDS is operating at its average rate or better, there will be no delay.

The best case: on the best day of the year in Airport A or B, when the flights are at their minimum "typical" occupancy, there will be no delay, no matter what rate the EDS is operating at.

1.4 Exploring τ : The effects of peak severity on τ , the mutability of τ

As we show in Appendix B, based on our assumptions and on empirical data, the number of non-peak hour bags, related to B_{peak} by γ_k (where k is the number of hours after peak and –k is the number of hours before peak), are most likely going to be low enough not to impinge on our ability to use $T = 1 + \tau - \mu$ hours to process B_{peak}. This means that

$$\gamma_k \equiv \frac{B_{peak+k}}{B_{peak}} \le \frac{1}{1 + \tau - \mu}.$$

The 1 in the above equation has units of hours. We cannot hope to change the daily flight schedule to remove peaks at certain hours; thus, peak hours and γ_k values are out of our control. However, how can we optimize our model to account for higher and lower γ_k coefficients?

Higher γ_k

Momentarily assuming an even distribution of bags over multiple hours, the rate $r_{m,n}$ of bag processing is the number of bags to be processed divided by the time, where *m* is the number of hours before the peak hour, and *n* is the number of hours after the peak hour. The rate needed to process the peak hour's bags and the bags from the previous hour is the total number of bags divided by the total time:

$$\frac{B_{peak-1} + B_{peak}}{1 + (1 + \tau - \mu)} = r_{1,0}$$

$$\frac{(1 + \gamma_{-1})B_{peak}}{2 + \tau - \mu} = r_{1,0}$$

Generalizing for all hours before and after peak hour, we get

$$\frac{B_{peak} \sum_{k=-m}^{n} \gamma_k}{n + m + (1 + \tau - \mu)} = r_{m,n},$$

that is, the minimum rate of EDS output needed to process all bags in one day, where *m* is the number of hours before peak and *n* is the number of hours after peak. Therefore, including the actual peak hour, the total hours of flight operations is $n + m + 1$. Thus, assuming that Airports A and B are not traveling close to the speed of light, $n + m + 1 \le 24$ hours. In practice, many airports partially or fully close for a few hours each day after midnight; this separation allows us to treat all days separately.

We cannot, however, in practice assume even distribution of bags over the entire day; hence, the existence of peak hours. When γ_k is relatively large, i.e. greater than the inverse of $1 + \tau - \mu$, the quantity of bags B_{peak+k} will strain the EDS system in the same way that B_{peak} does, limiting the amount by which we can level the peak.

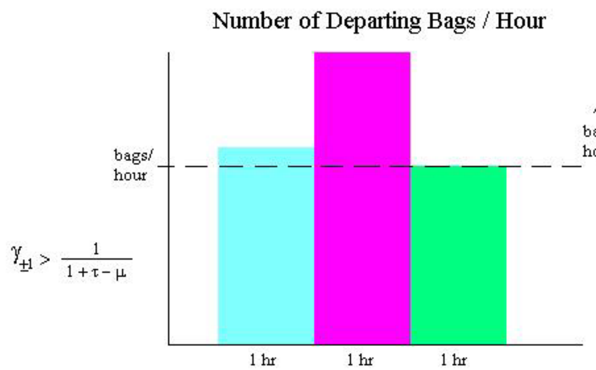


Figure 1a

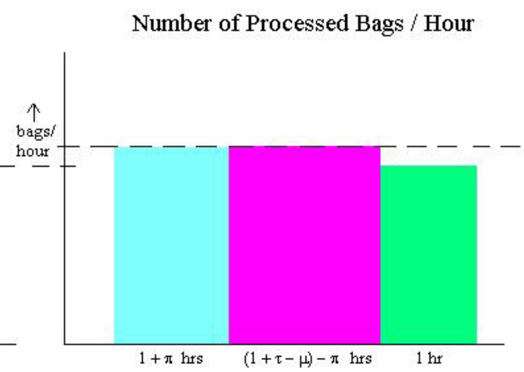


Figure 1b

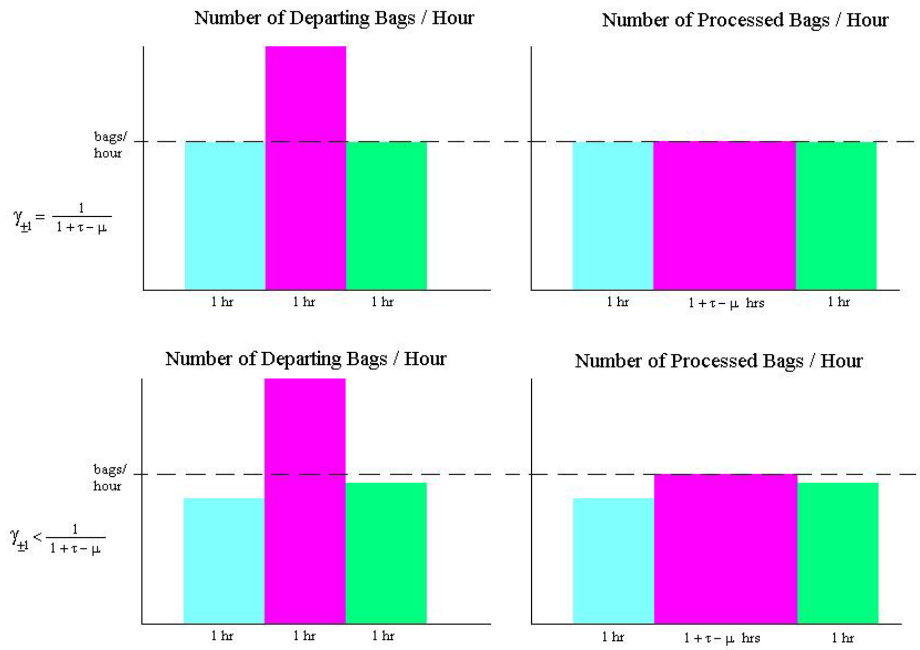
As the diagram shows, since the hour before peak requires a greater EDS throughput rate than

$$r_{0,0} = r_{peak} = \frac{B_{peak}}{1+\tau-\mu},$$

we must increase the rate to $r_{1,0}$, thus increasing the number of EDS needed. The period before peak subsequently appropriates some of $\tau - \mu$. Thus, increasing τ under these conditions yields more sharply diminishing returns, as the mandatory early arrival time becomes increasingly shared with the other periods. This makes intuitive sense: the less significant a peak, the less special attention it deserves, just like a mediocre middle sibling.

This is a very important fact: at airports with less acute peak hours, passengers should not be forced to arrive unnecessarily early, reducing wait time – something that will make air travelers happy. Less waiting time reduces the total time of a flight, thereby lowering a potential passenger's opportunity cost to fly. This should result in more air customers, thus increased revenue for airlines – an important consideration, especially in the post-September 11 market. Consequently, increased numbers of passengers would strain the EDS system already in place. This equilibrium-seeking effect, although interesting, will not be included in this model. Nevertheless, the potential benefits of choosing a proper τ have been noted.

Lower γ_k



Figures 2a, 3a　　　　　Figures 2b, 3b

Under these conditions, the full $1+\tau-\mu$ can be used by the peak period. A relative peak still exists; the lower the γ_k, the greater the peak. So, we can increase τ until

$$\gamma_k = \frac{1}{1+\tau-\mu},$$

generating a plateau on the lowest possible $r_{m,n}$ value, thus minimizing the necessary number of EDS. Of course, raising τ past a certain point, even if γ_k is still less than the inverse of $1+\tau-\mu$, is not good: just as passengers love shorter wait times, they also become riotous when forced to wait for hours.

Other considerations

However, τ is less mutable than we would like. Mandatory early arrival times are generally set by the airlines, not the airports. While airlines do have a vested interest in choosing an appropriate τ, the threat of a free rider problem exists. That is, with a τ of one hour, for example, an airline could cut their τ to 30 minutes, and thanks to our baggage prioritizing system, in which bags are sorted for processing according to the proximity of their flight's departure time, reap the benefits of a speedy EDS inspecting regime while appearing more attractive to air customers due to its short wait time. This possibility puts pressure on other airlines to cheat the system and do the same, bogging the EDS process down and causing delays. We, however, are optimistic in the ability of the major airlines, under the advice/coercion of the TSA, to collude and minimize this risk.

Another problem is that of standardization. Even if we can optimize the τ for our airports, we need to consider the rest of the airports in the country. A national system in which all airports require passengers to arrive at different early times would cause great confusion for travelers. Add to that the problem of quantization, that times are generally expressed in rounded, specific numbers, such as 30 minutes, 45 minutes, etc. We cannot tell people to show up 43.68 minutes early for a flight. Thus, some pragmatic sacrifices to efficiency will need to be made. As we have shown, assuming a 45-minute τ and relevant γ_k values in the acceptable range, we believe that our τ assumption is reasonable. Other values for our variables would affect the ideal τ, but optimizing τ should be a final consideration when applying the model to other conditions.

2. Task 2
2.1 Position Letter to Accompany Task 1

One of the airlines' primary security-related objectives is to prevent explosives from ever reaching the plane. In order to do this, one must have an effective system of explosive detection. The Federal Aviation Administration (FAA) has determined that the Explosive Detection System (EDS) is effective, and new laws have mandated the inspection of all checked bags with these machines. These machines, however, are costly.

We have developed a mathematical model to determine how best to implement the EDS system, maximizing security and minimizing the cost to taxpayers. Even though some of the variables in our model are solely within the domain of the airport's administration, there are several constraints under which the airlines must operate in order for our model to be optimized. For instance, the minimum early passenger arrival time is set by each individual airline (i.e. how long before departure the airline closes bag check-in, labeled τ in the model). If the quantity of EDSs remains constant, then by increasing or decreasing the minimum early passenger arrival time, the airline is increasing or decreasing, respectively, the time available to process the bags before each flight can take off.

Another variable that affects our model is the time that it takes for a bag to travel between the EDS and the airplane, labeled μ in our model. We do not know what power the airline wields over this variable, but if there is any possible way to decrease this time variable, then the overall model will be more efficient.

Cooperation between the TSA and the airlines is essential on these issues. Other than these, though, there is nothing that the airline can do that will directly affect the model's efficiency. However, the airline can work with the airport administration in optimizing other factors, so as to minimize delay for their own flights. Such variables might include obtaining more efficient machines (by possibly contributing money to the advancement of research in the fields of science, technology, engineering, and mathematics, with relation to explosives detection), or educating airport employees, so as to increase the efficiency of such variables as the travel time of a bag between the EDS and the airplane or the rate at which the EDS can process bags.

3.Task 3
3.1 The algorithm

As we discussed earlier, the ideal situation is when the flow of incoming and outgoing bags remains fairly steady. Therefore, since the number of bags depends on the number of passengers, the flights should be distributed such that the number of passengers checking bags and departing remains as constant as possible over the peak hour.

We developed the following algorithm to help airlines determine how to schedule the departure of different flight types within the peak hour so that the number of passengers, and, consequently, the number of bags, is evenly distributed. Assumptions put forth at the beginning of this paper hold true here as well:

```
1. Obtain data on the number of flights and seats on each flight
     during the peak hour.
2. Modify the seat data to represent the average number of people
     on each flight.  To do this, multiply by the estimated
     percent of seats filled for the type of the given flight.
     e.g. for a flight with 34 seats, multiply by 86.79%
```

```
3. Calculate total number of people on all flights during the
     peak hour.
4. Determine the desired number of time intervals during the peak
     hour.  We chose 6 as an appropriate number.
5. Determine the average number of people to fly during each time
     interval. Allocate that number of "spaces" for each interval,
     i.e. total number of people divided by 6
6. Do the following n times (where n = total number of flights):
     a. Find the flight with the most people on it.
     b. Starting at the first interval, and searching
     sequentially through to the last, find the time
     interval with the most number of "spaces" still
     available.
     c. Assign said flight to this time interval.
     d. Subtract the number of available "spaces" by the number
          of people on said flight.
7. Make sure there is a flight at :00 and :59 to ensure the efficiency
of our model, so as to maximize the time interval available for
processing and allow the constant use of our machines at full capacity,
thus preventing idleness, which is the devil's workshop.  To do this:
     a. For the first 30 minutes, start at the beginning of the
          time interval and evenly distribute the interval's assigned
          flights in order of decreasing flight capacity and
          increasing time.
     b. For the second half hour, start at the end of the time
          interval (:39, for instance) and evenly distribute
          the interval's assigned flights in order of
          decreasing flight capacity and decreasing time.
```

Essentially, we are evenly distributing the flights scheduled in this peak hour among six 10-minute intervals. The flights were modified to represent the average number of passengers per flight, rather than the number of seats per flight, since the former has more impact on the number of bags that will be scanned than the latter. The manner in which the flights were distributed among those intervals is analogous to filling a jar with different-sized rocks. One begins by adding the largest rocks, then smaller rocks, then pebbles, then sand, and finally water. With each additional step, you are filling in gaps. If you start with water and fill up the jar, then there is no room left for anything else. Thus, we start with the larger capacity flights and move our way down.

Please see Appendix E for a computer program written in C++ that implements the above algorithm, and thus proves that this algorithm works. This program was run using data from Airports A and B. The exact output for Airport A can be seen in Appendix E, but the summary for the output of Airport A follows.

Table 3

Flight Interval:	Type 1 Flights	Type 2 Flights	Type 3 Flights	Type 4 Flights	Type 5 Flights	Type 6 Flights	Type 7 Flights	Type 8 Flights
:00->:09	2	0	1	0	3	0	0	1
:10->:19	2	1	0	1	3	0	1	0
:20->:29	2	1	0	1	2	2	0	0
:30->:39	1	0	1	0	4	1	0	0
:40->:49	1	2	0	0	4	1	0	0
:50->:59	2	0	1	1	3	1	0	0
Total	10	4	3	3	19	5	1	1

Now that we have the flights distributed among the 10-minute intervals, we can evenly distribute them within each interval. A potential schedule follows:

Table 4

	Flight Type by Minutes (such that :x1 = :01, :11, …, :51)									
Flight Interval	:x0	:x1	:x2	:x3	:x4	:x5	:x6	:x7	:x8	:x9
:00->:09	8	5	5	5	3	1	1			
:10->:19	7	5	5	5	4	2	1	1		
:20->:29	6	6	5	5	4	2	1	1		
:30->:39				1	3	5	5	5	5	6
:40->:49			1	2	2	5	5	5	5	6
:50->:59			1	1	3	4	5	5	5	6

The following graph shows the number of bags still left for the EDS to process after each minute in Airport A, as a product of time:

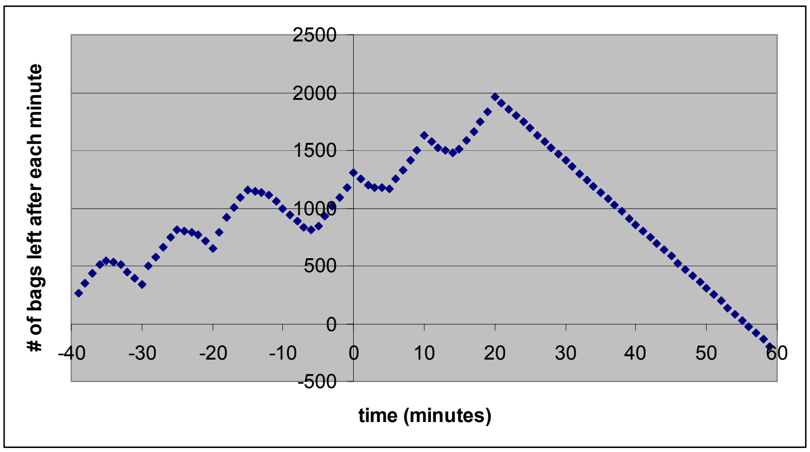


Figure 4

Since the preceding graph is fairly linear & positively increasing until the bags stop coming in, and never dips down into the negative values for x until the end of the peak hour (here, defined from 0 to 59 minutes), we can accept this graph as proof that the algorithm works for Airport A.

To justify this assertion, we propose some intuitive counterexamples. Suppose all flights depart at the same time; thus, all bags arrive at the same time. EDS operators only have 39 minutes to process all bags, and condensing the busiest hour of work into only 39 minutes would lead to excessive capital outlay for unnecessary EDSs. Now let us suppose that all but one flight occur in the first half hour of the peak, with bags evenly distributed over the period, and the final flight occurs at the last minute. Therefore, we have the 45 minutes early arrival time from τ, plus the 30 minutes of flights, minus our 6 minute μ to process the first half hour's bags. This is a total of 69 minutes to get the majority of the bags processed. We would then have a remaining 30 minutes to process the bags for the lone last flight. Unless this flight has a huge number of bags on it, such that the bags per minute that need to be processed equals the bags per minute that our machines are capable of, our machines will not be running to their fullest capacity during this last half hour. The most optimal set-up is if the machines are running at their fullest capacity during the entire processing time, which can only be ensured if the flights are spaced out such that the rate of processing remains fairly steady throughout *T*.

For Airport B, the summary of the distribution of flights among the six 10-minute intervals follows:

Table 5

Flight Interval	Type 1 Flights	Type 2 Flights	Type 3 Flights	Type 4 Flights	Type 5 Flights	Type 6 Flights	Type 7 Flights	Type 8 Flights
:00->:09	1	1	1	1	1	1	0	1
:10->:19	2	2	0	1	2	1	1	0
:20->:29	1	1	2	0	2	1	1	0
:30->:39	2	0	2	1	0	3	0	0
:40->:49	1	1	1	1	2	2	0	0
:50->:59	1	1	1	1	2	2	0	0
Total	8	6	7	5	9	11	2	1

A similar table for the minute-by-minute schedule for Airport B follows:

Table 6

	Flight Types by Minute									
Flight Interval	:x0	:x1	:x2	:x3	:x4	:x5	:x6	:x7	:x8	:x9
:00->:09	8	6	5	4	3	2	1			
:10->:19	7	6	5	5	4	2	2	1	1	
:20->:29	7	6	5	5	3	3	2	1		
:30->:39			1	1	3	3	4	6	6	6
:40->:49			1	2	3	4	5	5	6	6
:50->:59			1	2	3	4	5	5	6	6

The following graph shows the number of bags still left for the EDS to process after each minute in Airport B, as a product of time:

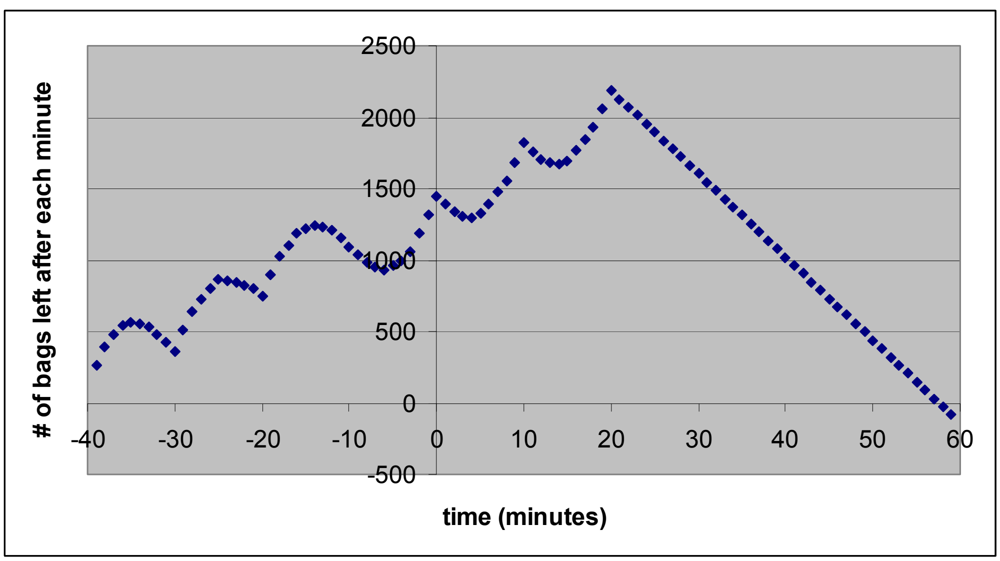


Figure 5

Since the preceding graph is fairly linear & positively increasing until the bags stop coming in, and never dips down into the negative values for x until the end of the peak hour (here, defined from 0 to 59 minutes), we can accept this graph as proof that the algorithm works for Airport B. (Justifications for this are presented with the graph shown for Airport A.)

Thus one has the proven minute-by-minute schedules for the peak hours of both Airport A and Airport B, and one can easily apply this algorithm to the peak hours of other airports.

(Side Note: We welcome you to visit our response to Task 6, where we have explored the possibility of having to schedule more than 60 flights in one hour.)

4. Task 4
4.1 Memorandum to Mr. Sheldon

Memorandum

Date: February 10, 2003

To: Mr. Sheldon and Airlines

From: The Analysis Team

RE: Task 4: Recommendations on Checked Baggage Screening for Flights During the Peak Hours at Airport A & Airport B

Priority: High

Because the EDS machines are so expensive, we have aimed to find a way to minimize the number of machines needed so that the cost to the airports is reduced. Our model and cost function show that there are several ways to minimize needed machines and maximize efficiency.

First, operator training is crucial, not only in effective explosive recognition but also for time-efficient bag processing. There obviously is a limit on how fast the EDS machine can work, processing a certain number of bags per hour. However, operator inefficiencies can also slow this down, so if the rate of bags processed per hour were to increase through an increase in training, the money saved from having to buy another machine will surely be worth it.

In addition, by maximizing the amount of time we have to process peak hour baggage, we minimize the number of needed machines. Therefore, a recommendation of our model is to require passengers to check their baggage no later than 45 minutes before the flight. Currently, the standard for arrival time is between 30 and 45 minutes before the flight, but in the light of the TSA's requirements for 100% baggage screening, requiring an arrival time of 45 minutes seems reasonable.

Related to the amount of processing time available is the amount of time that must be reserved for transporting the baggage from the EDS machines to the airplanes. If this time can be minimized, for example using an efficient system of conveyor belts, than more there will be increased time to process peak hour baggage.

One of our more significant recommendations that our model puts forth, a recommendation that we have assumed to exist in this model, is the idea of prioritizing baggage as they come in. For example, if one person arrives 2 hours ahead of their scheduled flight, and another person arrives at the same time, but their flight is in 45 minutes, then the latter person's checked bags will be processed by an EDS before the former's. We believe that it is cost-effective to pay someone to prioritize the bags since it is cheaper to pay a few people to prioritize bags than to buy another machine to cover such inefficiencies.

Finally, our most significant recommendation on the checking of baggage is the aim for a steady flow of incoming and outgoing bags during the peak hour. By doing this, we ensure that the maximum capacities of the machines are used during the entire peak hour. This can be accomplished by distributing the flights such that the number of incoming passengers for departing flights, and hence the number of incoming bags, remains steady over the entire peak baggage processing time.

6. Task 6

6.1 Memorandum to the Director

Memorandum

Date: February 10, 2003

To: Director of the Office of Security Operations, Transportation Security Administration

From: The Office of Mr. Sheldon, the Director of Airport Security for the Midwest Region

RE: TASK 6: The Adaptation of a COMAP Model to all 193 Airports in the Midwest Region

Priority: [Urgent]

The algorithm presented in the model (under Task 3 of paper) can be easily adapted to all of the 193 airports in the Midwest Region. This memo will explain how this adaptation can occur.

The algorithm takes into account peak hours with less than 60 flights. However, it is possible to have more than 60 flights in this hour, if the airport is extremely busy. In this scenario, there would be more than 10 flights in each 10-minute interval. If this occurs, then an easy fix to the algorithm presents itself: during the scheduling of the flights in the minute-by-minute schedule, when each minute in the 10-minute interval already contains one flight, start over at the last minute, and work your way down. It is done in reverse, since the larger flights will already be at the beginning of the interval. Thus, this manner will balance out the number of passengers over the interval, more so than not working in reverse. Of course, if there are more than 20 flights, then one would start at the beginning again after working back to the first minute. Case in point, if there are 15 flights in the first 10-minute period, then distribute the first 10 flights so there is one flight every minute, :00 through :09. Next, distribute the remaining 5 flights so there is one flight every minute, :09 back through :05. Another case would be if there are 23 flights in the 10-minute period from :30 through :39. For this case, you would distribute them as above: distribute the first 10 flights so there is one flight every minute, :30 through 39. Next, distribute the second 10 flights so there is another flight every minute, :39 back through :30. Finally, distribute the remaining three flights so there is a third flight every minute, :30 through :32.

As shown in the equations expounded on elsewhere in this paper, there is a direct, linear relationship between the number of bags and the number of machines. Thus, when the number of bags is increased due to increased traffic at a given airport, the number of machines will be directly increased. For example, if at a certain airport, there are B bags and Q_{EDS} machines, and at another airport, there are 1.6*B bags, then the latter airport would require 1.6* Q_{EDS} machines. The same holds true if the number of bags is decreased, due to less traffic at a given airport. In this case, one would use the same idea, but the factor would be between 0 and 1.

If there is an increased μ at a specific airport and all else remains equal, then the τ should be increased equally to counteract the effects of the increased μ. This allows the total time to remain constant. An airport should pay attention to this if the time it takes to transport a bag from the EDS to the plane is significant.

7. Task 7

7.1 Cost analysis of EDS and ETD

$$C(\alpha,\omega,Z) = B_{peak}\left(\frac{\alpha(1000+c_i+\omega Z)}{\Omega_{EDS}\ell_{EDS}(1+\tau-\mu)}+\frac{(1.2-\alpha)(45+10\omega Z)}{\Omega_{ETD}\ell_{ETD}(1+\tau-\mu)}\right) \qquad \textit{(eq. 4)}$$

$C(\alpha,\omega,Z)$ = total cost of recommended system, as a function of α, ω, and Z
B_{peak} = total number of bags during the peak hour
α = percentage of B_{peak} that the EDS will screen
ω = operational cost of the EDS per hour, operational cost of the ETD machine is 10 times this amount.
Z = years
c_i = installation cost of EDS, dependent on airport, in thousands
ℓ = throughput rate of each machine, in terms of bags/hour/machine
Ω = percent of time that the machines are operational (given constants)
τ = minimum early passenger arrival time, in hours
μ = travel time of one bag between EDS and the plane, in hours
1000, 45 = buying cost of EDS and ETD machines, respectively, in thousands

Assumptions: In addition to our previous assumptions, we also assume here that the installation cost of the ETDs are negligible.

7.1.1 Deriving the Model

By requiring that 20% of all bags get screened through both an EDS and ETD machine, the effective number of bags to screen increases by 20%. The number of bags that will go through the EDS, B_{EDS}, plus the number of bags that will go through the ETD machine screening, B_{ETD}, must equal this effective number of bags. Therefore,

$$B_{eff} = 1.2B_{peak} = B_{EDS} + B_{ETD} \qquad \textit{(eq. 5)}$$

The time that the airport has to screen all these bags remains the same as in our previous model, and therefore, τ and μ have the same values as given earlier. (Although, Lao Tsu tells us that "The Tao that can be named is not the eternal Tao", whereas in our model, τ is both known and finite.[8]) Likewise, the equations to determine the number of EDSs will remain the same as it was in our previous model, and the number of ETD machines can be determined similarly.

$$Q_{EDS} = \frac{B_{EDS}}{\Omega_{EDS}\ell_{EDS}(1+\tau-\mu)} \qquad Q_{ETD} = \frac{B_{ETD}}{\Omega_{ETD}\ell_{ETD}(1+\tau-\mu)} \qquad \textit{(eq.6, eq.7)}$$

So what does it cost?

To determine the number of machines that would be optimal in each airport, we examine the effect that the quantities have on cost. The initial cost of each machine will equal the cost to buy each machine plus the cost to install it. EDSs are given as costing \$1 million, while ETD machines are only \$45K. Luckily, ETD machines are usually fairly small and portable, so their installation costs are assumed to be negligible. However, the installation cost of EDSs, c_i, is substantial: \$100K for airport A and \$80K for Airport B.

In addition to the fixed cost of buying the equipment, however, the variable cost of operating the machinery is significant in our cost equation as well. The variable cost per year is equal to the yearly operational cost of each machine, ω. The ETD machine is given as requiring 10 times the operational cost of the EDSs, or 10ω. Because this variable cost is dependent on time, the total cost at any given future time will depend on the number of years that the system has been running, which we designate here as *Z*. Therefore, the variable cost of each machine equals the operational cost time the number of years.

The total cost of our model, C(*Z*), is the fixed cost plus the variable cost of each machine. All costs in the following equations are given in thousands.

$$C(\omega, Z) = Q_{EDS}(1000 + c_i + \omega Z) + Q_{ETD}(45 + 10\omega Z) \qquad \textit{(eq. 8)}$$

Substituting equations (6) and (7) into the above equation, we get:

$$C(\omega, Z) = \frac{B_{EDS}(\$1{,}000{,}000 + c_i + \omega Z)}{\Omega_{EDS}\ell_{EDS}(1+\tau-\mu)} + \frac{B_{ETD}(\$45{,}000 + 10\omega Z)}{\Omega_{ETD}\ell_{ETD}(1+\tau-\mu)} \qquad \textit{(eq. 9)}$$

However, we know that the number of bags going through each EDS is related to the number of bags going through each ETD machine by equation (4). In addition, the number of bags going through each EDS is between 20% and 100% of the total number of peak hour bags. We represent this relationship by the coefficient α, such that $0.2 \le \alpha \le 1$.

$$B_{EDS} = \alpha B_{peak} \qquad \textit{(eq. 10)}$$

Substituting equation (10) and equation (8) into the cost equation, we are left with the important equation below.

$$C(\alpha, \omega, Z) = B_{peak}\left(\frac{\alpha(1000 + c_i + \omega Z)}{\Omega_{EDS}\ell_{EDS}(1+\tau-\mu)} + \frac{(1.2-\alpha)(45 + 10\omega Z)}{\Omega_{ETD}\ell_{ETD}(1+\tau-\mu)}\right) \qquad \textit{(eq. 11)}$$

[8] Tao Te Ching by Lao Tsu

Due to my practical knowledge of botched calculations, it is important to check that our units work. Remember that α is dimensionless:

$$\$ = bags\left(\frac{(\$ + \$ + \$/year * years)}{bags/hour * hours} + \frac{(\$ + \$/year * years)}{bags/hour * hours}\right) \qquad \textit{(eq. 12)}$$

As you might have noticed, our cost equation is a function of α, ω, and Z, since these are all variables that we will explore later. We know the throughput rate of both machines, the required check-in time of the passengers, and the EDSs cost of installation. We also know the number of bags during the peak hours. When applied to equations (6) and (7), B_{peak} directly relates to the number of EDS and ETD machines that the airport will have and will therefore tell us how many of each machine we will need, once we decide on an appropriate α.

Using Maple, we plot equation (11) as a function of α and keep ω constant. (Here, we arbitrarily assume ω = \$50K). We can see in Figures 6a and 6b that after various number of years, the cost of the machine can significantly depend on the number of bags that go through each machine, which depends on α.

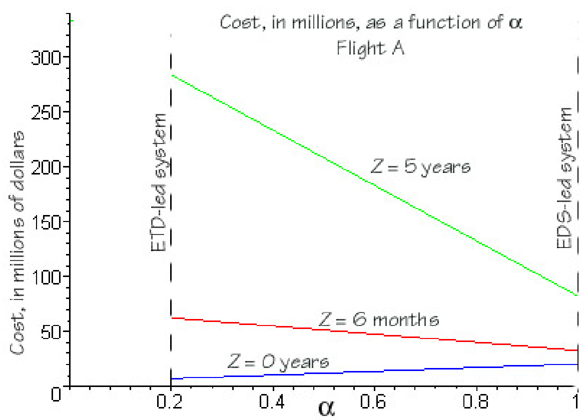


Figure 6a

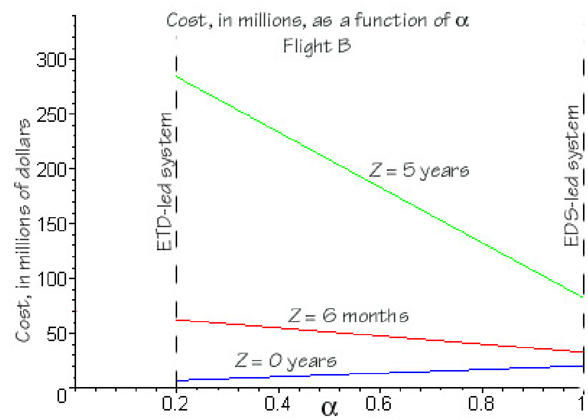


Figure 6b

Figures 6a and 6b demonstrate a crucial point: the function C(α,ω,Z) is linear, and the number of years, Z, affects its slope.[9] Except for the particular Z value that makes the slope = 0, the first derivative test shows that only 0.2 and 1, the extreme values for α, can yield minimum values for C. This means that there are two significant cases to study:

(1) the EDS-led system, in which EDSs are the first tier of baggage scanning, processing 100% of the bags, and ETD machines are the fail-safe, scanning 20% of the bags; or, vice-versa,

(2) the ETD-led system, in which ETD machines process 100% and the EDSs scan 20%.

The third case, when α is somewhere between these two extremes, will later be briefly discussed, and shown undesirable.

As one can see, installing an ETD-led system (i.e. α = 0.2) would be beneficial only during the first few months. This makes sense since the installation cost, a fixed cost, of an all-EDS system is very expensive, while the accumulation of the high variable cost of operating the ETD machines is kept comparatively low during these first months. However, after a few months, it is optimal to have α at 1, or an EDS-led system, since this has a minimum cost as Figures 6a and 6b show.

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[9] If we had assumed a value for Z (number of years) and chosen multiple values for ω (operational costs), ω would determine the slope. We will return to this relationship shortly.

However, this graph assumes that the cost of operation of the EDS is \$50K per year, which may or may not be realistic. A different ω will affect the slopes of this graph, thereby affecting the time at which α = 1 becomes optimal. Therefore, by finding where the derivative of the Cost function is zero, we can find the critical turning point for our model at any ω, such that after this time, an EDS-led system would be more desirable than an ETD-led system. Therefore the following is the partial derivative of equation (11) with respect to α:

$$\frac{\partial}{\partial \alpha} C(\alpha, \omega, \mathrm{Z}) = B_{peak}\left(\frac{(1000 + c_i + \omega \mathrm{Z})}{\Omega_{EDS}\ell_{EDS}(1+\tau-\mu)} - \frac{(45 + 10\omega \mathrm{Z})}{\Omega_{ETD}\ell_{ETD}(1+\tau-\mu)} \right) \qquad \textit{(eq.12)}$$

When this equation is set to zero and solved for *Z* using cross multiplication, the following equation remains, a function of ω since all the other terms are constants.

$$\mathrm{Z}(\omega) = \frac{1}{\omega}\left(\frac{\Omega_{EDT}\ell_{EDS}(45) - \Omega_{ETD}\ell_{ETD}(1000 + c_i)}{\Omega_{EDT}\ell_{ETD} - 10\Omega_{EDS}\ell_{EDS}} \right) \qquad \textit{(eq. 13)}$$

Notice that ω is inversely related with Z(ω). Also notice that B_{peak} and (1+τ-μ) cancel out, thereby not influencing our model's critical cut-off time. Therefore, the only difference between Airport A and Airport B is the installation cost, which is unnoticeable when plotted. With the help of Maple, Z as a function of ω is plotted for Airport A, as seen in Figure 7.

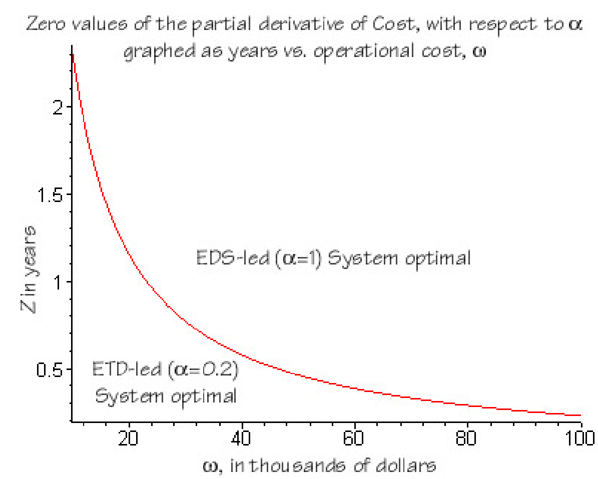

Figure 7

For (ω,Z) combinations on the curve, both ETD-led and EDS-led systems are equal in cost, but in practice this is a. For (ω,Z) combinations below the curve, an ETD-led system is most cost-efficient; however, an EDS-led system is more likely to be optimal since the yearly operational cost of each machine, including full-time wages for enough workers to operate each machine, will be high enough to make an EDS-led system cheaper in less than one year. Given not only a life expectancy of EDSs around 10 years[10] but also bureaucratic inertia, we cannot expect the EDS-ETD system baggage inspection system to be replaced soon enough so that an ETD-led system will minimize costs.

With all this information, it seems pretty clear that an EDS-led system is more desirable than an ETD-led system. However, a third case needs to be addressed as well. In this case, α is neither 1 nor .2 but something in between. Although this is not optimal in cost at any point in time, it is nevertheless a possibility

[10] International Security Systems, "The EXact"

for our model. However, momentarily ignoring cost, this system would not be practically beneficial either. In our other two cases, the airport would either be predominantly EDS or ETD machine with only 20% of the bags going through the other machine. This means that whichever machine is predominant would be considered first tier (or at least at a higher tier than the secondary machine), and therefore all bags would pass through this tier first, and only pass to the second tier after an alarm has sounded or a bag is randomly chosen for secondary screening. However, this third case would mean that both these machines are on the same tier, making for a more complex system of screening. For example, with half the bags going to ETD machines and half the bags going to EDSs, operators would have to have some efficient method arranged so that those bags that sound alarms on one machine could be more carefully checked by the other without getting lost among its other bags.

However, even though ETD machines become quite expensive after a short amount of time because of its high operational cost, its low fixed cost might come in handy in dire circumstances. As mentioned earlier in the paper, airports might run into delays during the peak hours of peak times of the year, such as around Thanksgiving. It would not be cost-efficient to buy extra EDSs just to handle these periods, however airports could buy extra ETD machines. These machines could be stored and not used unless really needed, thereby saving on operational costs during much of the year. However, determining the number needed and the actual efficiency of this suggestion, one would need to analyze the data regarding number of bags during these times of the year.

Determining Q_{EDS} and Q_{ETD}

After all this, we have finally determined that α should be set to 1, in other words 100% of the bags will go through an EDS. Therefore, we can calculate the total number of machines to buy by plugging the numbers into our initial equations. Here, we use a combination of equations 1, 2, and 6.

$$Q_{EDS} = \frac{\alpha B_{peak}}{\Omega_{EDS}\ell_{EDS}(1+\tau-\mu)} \qquad Q_{ETD} = \frac{(1.2-\alpha)B_{peak}}{\Omega_{ETD}\ell_{ETD}(1+\tau-\mu)} \qquad \text{(eq. 14, eq. 15)}$$

In order to apply this model to our two large, Midwestern airports, we estimated $\ell_{ETD} = 47$ bags/hour/machine. The reason we chose this number is because this was the average throughput rate of the ETD machines at the Summer Olympics in 2002.[11] The other constants are the same values as we used in our earlier model:

$$B_{peakA} = 5286$$
$$B_{peakB} = 5683$$
$$\ell_{EDS} = 190$$
$$\Omega_{EDS} = 0.92$$
$$\Omega_{ETD} = 0.98$$
$$\tau = 0.75$$
$$\mu = 0.1$$

Using these values, we calculate:

$$\boxed{\begin{aligned} Q_{EDS_A} &= 18.33 \approx 19 \\ Q_{ETD_A} &= 13.91 \approx 14 \end{aligned}} \qquad \boxed{\begin{aligned} Q_{EDS_B} &= 19.70 \approx 20 \\ Q_{ETD_B} &= 14.96 \approx 15 \end{aligned}}$$

Note that, as should be expected, the Q_{EDS} values for each airport are unchanged from previously when we had not yet considered the ETD machine.

[11] Viggo Butler & Robert W. Poole, Jr. "Rethinking Checked Baggage Screening."

7.2 Memorandum to the Directors

Memorandum

Date: February 10, 2003

To: Director of Homeland Security

CC: Director of the Transportation Security Administration

From: The Office of Mr. Sheldon, the Director of Airport Security for the Midwest Region

RE: Task 7: Technical Analysis of the Advanced Screening Policy, Using Both EDSs & ETD machines

Priority: High

Due to additional security methods that require up to 20% of checked baggage to be screened through two systems, that is, both EDS and ETD machines, our model is slightly revised. The number of machines that would be required to process the bags is directly related to the number of peak hour bags, as before. However, we now have the factor of cost to consider, which will influence how many machines of each type to purchase.

After analyzing the cost equation that incorporated the purchase costs, installation costs, and operational costs per year of each machine, we came to the following conclusions. During the first few months after the new system is implemented, it is cheaper if the majority of the machines are ETD machines. In other words, 100% of the bags would be screened using ETDs and only 20% would be screened using the EDSs which would probably be considered second-tier screening (or at least a level below that of the ETDs). This is because the ETDs have a much lower initial cost than the EDSs. However, as these machines are probably expected to last longer than just a few months, this system is probably not desired. Therefore, the second option is to have 100% of the bags screened through the EDS machines, and only 20% through ETDs, which is the most cost effective method after only a few months. This is due to the extremely high operational costs of the ETDs. With this system, EDS machines would be the primary tier, while ETDs would be the secondary, used for rescanning random bags and bags that sound alarms.

Purchasing enough EDS machines to screen 100% of the peak hour bags will have an initial cost of several million dollars. However, such a policy is easily justified in the light of the millions of dollars that airports would save over the next few years in operational costs.

8. Task 8

8.1 Recommendations for future funding

Although it is clear that an EDS-led system, with merely enough ETD machines to cover 20% of the bags, is optimal based on our calculations, it might not be the absolute best solution. An important consideration is whether or not new technology might be able to replace the machines before the critical cut-off time, as given in Figure 7. For example, if current technology trends show that a better baggage screening system will be ready in less than a year, it might be worth taking the risk and buying an ETD-led system. Then, within the year, buy the better machines, with lower operational costs, that can replace the ETD machines. However, not only would this save very little, but this is quite a risk to take since your operational cost for the ETD machines will hurt the airport terribly if better technology does not come out in time. Therefore, our model shows that unless current trends show an immediate market introduction of new and advanced technology, the best solution for now is to have all bags screened by the EDSs, and only 20% of the bags are screened by the ETD machines.

Down the road, however, we may need to re-evaluate the system. Significant advances in the field of baggage detection devices *are* currently underway, and information can be found regarding the other possibilities that will probably emerge within the next few years. Some of these new technologies are predicted to have increased accuracy rates, lower false positive rates, and higher speed capacities. There is little available documentation regarding when such machines will be put on the market, but we can assume that in a few years, it might be beneficial to replace or significantly modify the system we are currently recommending. How can we determine the conditions under which we should change baggage inspection systems?

We must examine the variable costs involved. Previously we have said that this means the operating costs, but now we want to consider the more distant future. We need to account for the fact that we will have to repurchase the machines eventually. Let ψ be defined as the life expectancy of a machine. Factoring it into the cost function from Task 7, we get

$$C(\alpha,\omega,Z) = B_{peak}\left(\frac{\alpha(\left\lceil\frac{\psi_{EDS}}{Z}\right\rceil(1000+c_i)+\omega Z)}{\Omega_{EDS}\ell_{EDS}(1+\tau-\mu)}+\frac{(1.2-\alpha)(\left\lceil\frac{\psi_{ETD}}{Z}\right\rceil 45+10\omega Z)}{\Omega_{ETD}\ell_{ETD}(1+\tau-\mu)}\right)$$

Thus, every ψ years, we need to repurchase and install machines. As we have chosen an EDS-led system, we are interested mainly in the life of the EDS, which we have found to be about 10 years.[12] Thus, at year ψ, we essentially start the cost function over, ignoring the sunk costs accrued over the previous years. We will be more likely to change machines at this point due to the high costs facing us.

Let us now generalize our models to incorporate variables for a new machine, machine X, assuming that the decisions regarding any other necessary machines, i.e. for re-screening, will be decided independently of our model.

$$\begin{aligned} Q_X &= \frac{B_{peak}}{\Omega\ell(1+\tau+\mu)} \\ C_X &= Q_x(c_0+\omega Z) \\ C_X &= \frac{B_{peak}(c_0+\omega Z)}{\Omega\ell(1+\tau+\mu)} \end{aligned}$$

The new variable that has not been previously introduced in this paper is c_0, the fixed cost of the machine (purchase cost plus installation cost). Lacking data for future machines, we can only discuss the effects of in a change, *ceteris paribus*, in some of the variables relative to the EDS values, to show the effects on the likelihood of adapting these new machines.

Clearly, one would want to keep the total cost, C_X, minimized. The rest of the exploration of these variables will examine that. To maintain a low overall cost, it is desirable that the purchase and installation costs are low. Likewise, the operating cost per year, ω, should be kept as low as possible.

Generalizing the current technology and research in technology that exists now, there are two major categories of screening methods. One category is that of large expensive machines that have a high throughput rate and low manual labor. EDSs fall under this category. The other category includes those machines that are fairly cheaper but require a large amount of manual labor and therefore a slower

[12] International Security Systems, "The EXACT"

throughput rate, such as ETD machines. As we showed in our previous model between EDS and ETD-led systems, in the long run high initial fixed costs often outweigh high operational costs. However, accurate comparisons between machine models and technologies are impossible without actual numbers regarding costs and efficiencies.

In addition, the rate of the number of bags per hour that each machine can process, ℓ, should be as high as possible. Increasing the rate, for example to between 1000 and 2000 bags per hour – which the Pulsed Fast Neutron Analysis (PFNA) machine is expected to be able to do[13] – can have a significant impact on cost because fewer machines will be needed. Likewise, the percent of time that the machine is operational, Ω. should be kept as high as possible as to avoid having to buy extra machines to account for downtime.

Other variables that we should heavily weigh are the false positive rate, the false negative rate, and the human reliability factor. The false positive rate and the false negative rate should both be kept as low as possible, but it is *more* important that the false negative rate be extremely close to 0, as this affects the accuracy of the machine, while the false positive rate merely affects the efficiency of the machine. Increased precision would not only increase the safety of our air traffic system but also reduce the number of secondary, fail-safe screening devices, thus saving money. Currently, EDSs are widely reported to have between 22 and 30% false positive rates, which is ridiculously high. New technology seems to be decreasing significantly this inefficiency, which will result in less required re-screenings and human intervention. A machine with high false negatives used as a first-tier scanner (as in the EDS in the EDS-led system) is very dangerous, and to counter the threat of explosives slipping through, costly random screening of negatives with a second device will be needed, though still not eliminating the said threat. In addition, as we just suggested, it is desirable to reduce a machine's reliability on human interaction and interpretation, for "To err is human."[14] Operators, much like the writers of this paper, have a natural tendency toward error by nature of their species and are more likely to miss detection of a hidden explosive than a highly accurate and specific machine, and, thus, the machine should rely as little as possible on human interpretation, instead operating automatically. Additionally, if fewer people are involved, the annual operating cost is slashed due to the reduced payout of wages.

Research

Logically, we believe that science, technology, engineering, and mathematics (STEM) research should be directed towards the development of machines that would increase both security in the aviation industry and the cost-efficiency of our model. Science research should probably be directed towards discovering more advanced ways to detect explosive materials, including contact with government intelligence on new explosive materials, so as to be able to detect them in a timely fashion. Technology research should go towards more automated methods. Engineering research should go towards creating a smaller machine that is more portable and thus less costly to produce and install. And the mathematics research should perhaps go towards creating a more complex model to include these factors. We suggest a sort of international competition of undergraduate students. Research should be conducted at such a pace so as to have a more effective and less costly alternative to the EDS-led baggage screening system *at least* before the EDS life expectancy expires, and new machines must be purchased.

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[13] Tsahi Gozani. "Hearing On Role Of Military Research"
[14] Benjamin Franklin, http://www.brainyquote.com/quotes/quotes/b/q136956.html

8.2 Addendum to memorandum from 7.2

Addendum to Memorandum

Date: February 10, 2003

To: Director of Homeland Security

CC: Director of the Transportation Security Administration

From: The Office of Mr. Sheldon, the Director of Airport Security for the Midwest Region

RE: Task 8: Future Scientific Research Programs & Funding

Priority: High

New technology will most likely not affect our model because, even though extensive research is occurring as we speak, no new technology will be ready for the market in the few months for us to consider anything other than an EDS-led system for the airports.

However, after a few years, a re-evaluation of the model may be necessary. We can therefore generalize the model so that all the factors that affect cost can be examined. In doing this, we find that technologies that have lower fixed and operational costs are recommended. Research into increasing throughput rate is recommended, as well, for this would significantly cut the number of necessary machines. In addition, technology that has better accuracy, both in lower false positive rates and in lower false negative rates, would mean better efficiency and safer travel. Finally, we recommend machines that are not as dependent on human interpretation, because doing so would both increase operational cost as well as lower the accuracy of machines in many cases.

Therefore, in conclusion, we recommend funding research that will increase security while not significantly increasing cost. Science research should probably be directed towards discovering more advanced ways to detect explosive materials, including contact with government intelligence on new explosive materials, so as to be able to detect them in a timely fashion. Technology research should go towards more automated methods. Engineering research should go towards creating a smaller machine that is more portable and thus less costly to produce and install. And the mathematics research should go to perhaps creating a more complex model to include these factors, possibly by funding more COMAP competitions. Research should be conducted at such a pace so as to have a more effective and less costly alternative to the EDS-led baggage screening system *at least* before the EDS life expectancy expires, and new machines must be purchased.

9. Conclusion: Strengths and Weaknesses

The main strength of our model is that, like an organized crime leader, it is very difficult to make accusations of wrongdoing stick. Throughout the paper, we have made an effort to show that the number of EDS machines determined by our model will work well even if some of the assumed constants and probabilities shift. More accurate statistical data, as should or could be available to airport administrators, would yield a more correct optimal number of machines needed. The delays caused by fluctuations in assumptions are, under most every case, within acceptable ranges for delay, i.e. delays for other reasons happening at the same time. If this model is implemented, it should be stressed that the system is designed so that no extra delays should be expected. If this argument is sold to the people convincingly enough, instances of delay should not make passengers more likely blame the EDS system over other causes for delay, such as waiting for connecting passengers, bad weather, or mechanical difficulties. We already showed that extreme circumstances, such as holiday travel days, normally experience delays, and any delay in the EDS system for that day, if not compensated with temporary ETD machines, would run parallel to the delays already occurring in the airport, not in addition to. Besides, air travelers will be willing to wait a few extra minutes occasionally if it gives them a sense of security that many lacked following September 11.

One weakness of our model is that we did not go into different methods for implementing the prioritization and queuing regime for bags entering the explosives scanners. We considered several options. One is that the tags placed on the bags at the check-in desk could list departure time on it, thus allowing easy sorting. This, however, does not allow for changes in departure time due to delays. Another idea was to have a departure listing screen, like those posted throughout the airports for passengers, displayed by the EDS machines. This list will be very long at a large airport, though, and would require EDS operators to recheck the display frequently. These ideas should be considered in the implementation of this model

Another weakness is that we ignored the placement of the EDS machines. We read and have seen that most EDSs are placed in the airport lobby near the check-in area. In a large airport, this could mean that the machines are spread out over a large area. So, the EDS machines could not work together like one unit, as our model implicitly assumes. This would mean a loss of efficiency: machines at one end of the airport could run out of bags while those at the other end could have too many. This problem could be remedied in the flight scheduling process, factoring in airline check-in desk placement in the even distribution of bags over the hour. The scope of that undertaking is far outside what we can accomplish here, though it ultimately deserves consideration.

10. Appendices

10.1 Appendix A

Technical Information Sheet (TIS) [15]

Peak Hour Flight Departures for Airports A and B

Flight Type	Number of Seats on Each Flight	Airport A Number of Flights of Each Type	Airport B Number of Flights of Each Type
1	34	10	8
2	46	4	6
3	85	3	7
4	128	3	5
5	142	19	9
6	194	5	10
7	215	1	2
8	350	1	1

Table 6

10.2 Appendix B

Determine the effects of non-peak hours on the maximum output rate and use empirical data to prove its inapplicability to observed conditions

By definition, the quantity of bags in the peak hour, B_{peak}, is greater than the quantity of bags in any other hour. We can represent the quantity of bags in other hours as a proportion γ of the peak bags:

$$B_{peak+k} = \gamma_k B_{peak} \; : 0 \le \gamma < 1$$

where *k* is an integer denoting hours before or after the peak hour. Therefore, $\gamma_0 = 1$, since the peak hour is *k* = 0. Our initial relation in the model shows that $r_{peak} = 0.92\ell Q_{EDS}$. At the peak hour, when no other hours matter,

$$r_{peak} = \frac{B_{peak}}{1+\tau-\mu}$$

That is, the total number of bags in the peak hour divided by maximum available time, yielding the rate of EDS output needed under these conditions.

When we use all $1+\tau-\mu$ to process B_{peak}, we only have 1 hour in which to process all of B_{peak+k} for $k = \pm 1$. But, as long as $\frac{B_{peak+k}}{1} \le r_{peak}$ (where 1 is the number of hours), the rate of EDS output needed to handle the peak period will be sufficient to process B_{peak+k} in one hour. Now we do some algebra starting from that inequality:

[15] Copied from the problem description.

$$\frac{B_{peak+k}}{1} \le r_{peak}$$
$$\frac{\gamma_k B_{peak}}{1} \le \frac{B_{peak}}{1+\tau-\mu}$$
$$\gamma_k \le \frac{1}{1+\tau-\mu}$$

Recall that the 1's in the above equations have units of hours, making γ_k dimensionless, as it should be. Thus, as long as that inequality holds, r_{peak} is sufficient to process bags in all periods, and the Q_{EDS} determined by r_{peak} is the minimum necessary quantity of machines.

For our estimated τ and μ values, γ_k must be less than 0.606. By analyzing empirical data, we learned that both the highest morning and evening peak hours were sufficiently greater than the neighboring hours such that $\gamma_{\pm 1} \le 0.606$.[16] Thus, we can operate at maximum time, 1.65 hours, without fear of other periods' effects.

Theoretically, however, it is possible for γ_k to exceed 0.606. Later in Task 1, we will adjust our model to account for this possibility.

10.3 Appendix C

Methods of Discovery of Distribution of Seats Filled in a Given Flight

Data from: *(http://transtats.bts.gov/DL_SelectFields.asp?Table_ID=259)*

Data on all flights was taken from the "T-100 Domestic Segment" table in the "Large Air Carriers" database from the Intermodal Transportation Database. The data consists of all flights originating in the United States in the year 2002. Fields selected included "Passengers" ("Number of Passengers") and "Seats" ("Available Seats"). The data was sorted by number of seats in ascending order, and then by number of passengers in ascending order. The percentage of passengers/seats was found for each flight. The data was broken up into different sections, defined by the number of seats per flight specified by the Technical Information Sheet (TIS) in Appendix A of the problem description (that, is in sections of flights containing between 34 & 85 seats, flights containing between 128 & 215 seats, and flights containing 350 seats). For each section, the flights with occupancies (i.e. number of passengers divided by the number of seats) of less than the specified amount in the aforementioned TIS were removed from the analysis as not applicable. Thus, all that remains in the first section of 34 to 85 seat flights are those flights with 70% to 100% occupancy; all that remains in the second section of 128 to 215 seat flights are those flights with 60% to 100% occupancy; and finally, all that remains in the third section of 350 seat flights are those flights with 50% to 100% occupancy. The sections of flights were split up into 5% intervals. The amount of 5% was thought to be a good value, as to split up the sections into several, yet still manageable for analysis, intervals. The number of flights within each interval was found. Tables follow on the data:

[16] In fact, our empirical γ_1 and γ_{-1} are rather close to 0.606. This suggests that our selection of τ and μ was either well informed or merely providential. For the much touted data, Appendix D

Table 7 - Section 1:

Intervals (% Occupancy):	Frequency:
70% -> 75%	62
75% -> 80%	50
80% -> 85%	55
85% -> 90%	56
90% -> 95%	71
95% ->100%	113

Table 8 - Section 2:

Intervals (% Occupancy)	Frequency
60% -> 65%	166
65% -> 70%	150
70% -> 75%	151
75% -> 80%	146
80% -> 85%	138
85% -> 90%	184
90% -> 95%	162
95% ->100%	298

Table 9 - Section 3:

Intervals (% Occupancy):	Frequency:
50% -> 55%	2
55% -> 60%	2
60% -> 65%	2
65% -> 70%	0
70% -> 75%	0
75% -> 80%	1
80% -> 85%	1
85% -> 90%	0
90% -> 95%	0
95% ->100%	2

As one can clearly tell, the amount of flights with 350 seats is not high enough to extrapolate any generalized statement on how many seats will be filled on any given flight. So, we analyzed only the frequency distributions for the first two sections. The bar graphs created from the data above follow:

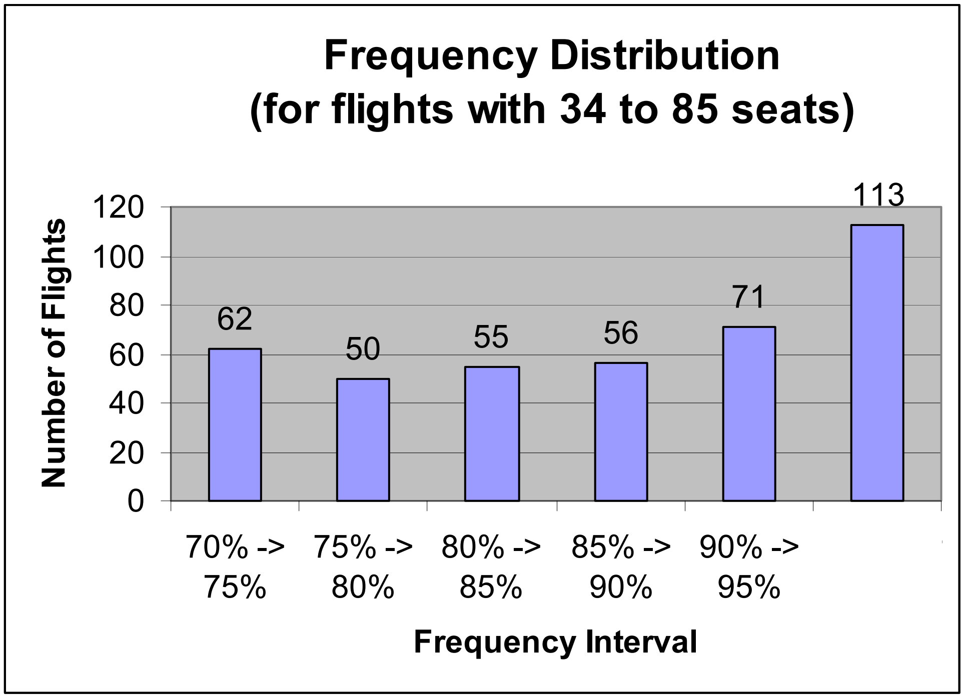


Figure 8

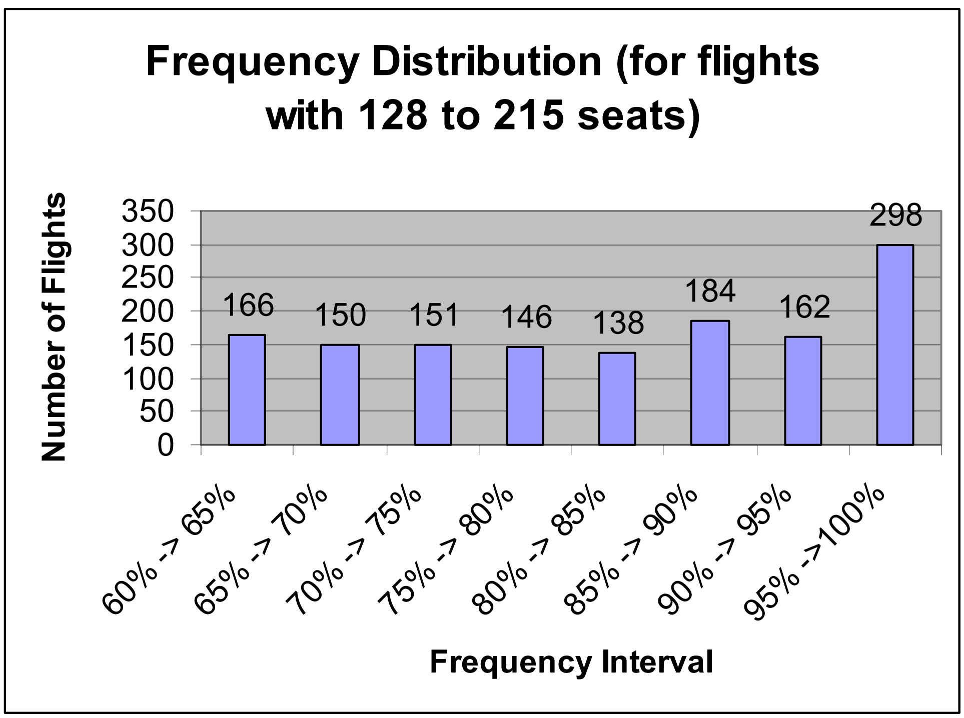


Figure 9

We thus generalize that the distribution is fairly linear up to the last interval of 95% to 100%, where the number of flights is around twice the number of flights in any previous interval. To return to the task at hand, the purpose of these discoveries is to be able to find the percentage of seats filled on a flight, so as to determine the number of bags on that flight, and eventually ascertain the number of machines needed to process those bags. Based on this data, we took the sum of the mean percentage values of each interval, added in an extra 97.5% (since there are twice the amount of flights during the interval of 95% -> 100%, this interval should be weighted twice as much). This sum is then divided by the quantity of the number of intervals plus 1, to obtain the average percentage value (the added "1" is due to the extra 97.5% added). This value is then used to determine the average number of bags per flight, etc.

We will now show how this is implemented into the three sections. The means of the intervals for the first section are: 72.5%, 77.5%, 82.5%, 87.5%, 92.5%, 97.5%, and then another 97.5%. The average of these values is about 86.79%. This number is then used for the average percentage of seats filled on a flight with 34 to 85 seats.

The means of the intervals for the second section are: 62.5%, 67.5%, in addition to the means of the intervals found in the first section. The average of these values is about 81.94%. This number is then used for the average percentage of seats filled on a flight with 128 to 215 seats.

Although we do not have good data on the flights with 350 seats, we can extrapolate and use the methods we used in the previous two sections. We thus assume that the distribution of seats filled on a 350-seat flight is fairly linear between 50% and 95% occupancy, with around twice the number of flights in the interval between 95% and 100% occupancy. Thus, the means of the intervals for the third section are: 52.5%, 57.5%, in addition to the means of the intervals found in the second section. The average of these values is about 77.05%. This number is then used for the average percentage of seats filled on a flight with 350 seats.

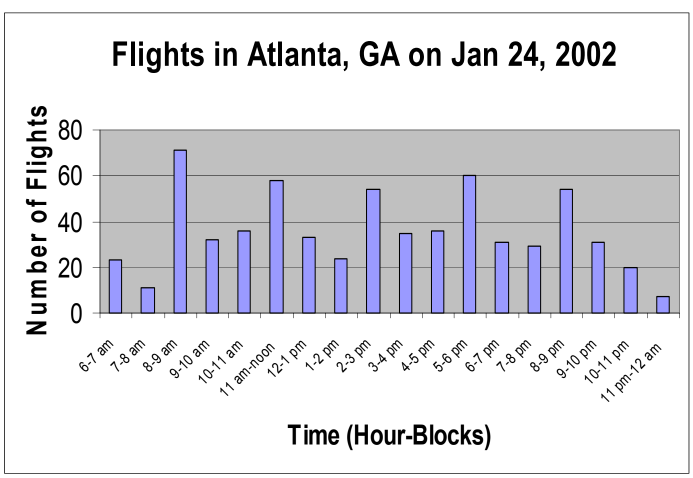
10.4 Appendix D

Methods of Discovery of Distribution of Flights within a Random Day of a Random Month of a Major City, spaced out among Hour-Long Intervals

Data from: *(http://transtats.bts.gov/DL_SelectFields.asp?Table_ID=236)*

Data on these flights was taken from "On-Time Performance" table in the "Airline On-Time Performance Data" database of the Intermodal Transportation Database. The data consists of all flights originating in Georgia in January of 2002. The data was sorted according to the date of flight, and then the departure time block (in hour-long intervals). The data was reduced to just flights originating in Atlanta, Georgia, so as to get specific data for one major airport. The final reduction was to a random day in January of 2002, in this case we used January 24th, and then confirmed our findings with the data from January 12th. The numbers were so close to identical that we assumed a certain regularity of flight schedules, which coincided with our personal flying experiences. Graphing the flights per hour yields the following:

Figure 10:



As the graph shows, there are several pronounced peaks, with the greatest peak being from 8-9 A.M. Focusing on this hour, we get the following values for γ :

Table 10

Hour block	Flights	γ	Hour block	Flights	γ
6-7 am	23	0.32	3-4 pm	35	0.49
7-8 am	11	0.15	4-5 pm	36	0.51
8-9 am	71	1.00	5-6 pm	60	0.85
9-10 am	32	0.45	6-7 pm	31	0.44
10-11 am	36	0.51	7-8 pm	29	0.41
11 am-noon	58	0.82	8-9 pm	54	0.76
12-1 pm	33	0.46	9-10 pm	31	0.44
1-2 pm	24	0.34	10-11 pm	20	0.28
2-3 pm	54	0.76	11 pm-12 am	7	0.10

Thus, the γ values around the peak are lower than our accepted 0.606. Even if the 5-6 P.M. hour is chosen as the peak, as it is a relative one, the neighboring γ values are right around .60, though not over. The relative peak hours could, if high enough and close enough, have an effect as well. However, assuming 8-9 A.M. as the peak again, we calculated that no relative peak is sufficient to affect the peak hour. Thus, we will proceed with assuming the irrelevance of other periods for now. In the last part of Task 1, we will consider other γ values and their possible implications on τ.

10.5 Appendix E

Computer Program Written Based on Algorithm to Distribute Flights In a Period of One Hour (Written in C++, with the g++ Compiler)

(Note: For this program, interval 0 refers to :00->:10 (of the hour), interval 1 refers to :10->:19, interval 2 refers to :20->:29, interval 3 refers to :30->:39, interval 4 refers to :40->:49, and interval 5 refers to :50->:59.)

```
#include <iostream>
int maxTimeInterval(float interval[]); //finds the maximum number in interval[] and
                                       //returns the index
double maxVal(double array[], int n); //returns the maximum value in given array
void convert(double array[], int n); //converts flights from having number of seats
                                     //available to average number of passengers on flight
int main()
{
    int n; //number of flights
    cout << "Enter total number of flights to be distributed: ";
    cin >> n;
    double flights[n]; //array for database of flights
    cout << "Enter the number of people on each of the " << n << " flights:\n";
    for(int i=0; i<n; i++)
        cin >> flights[i]; //get number of people on each of the n flights
    cout << "...thank you, please drive through to the next window\n";

    convert(flights,n); //converts flights from having number of seats available to
                        //average number of passengers on flight
```

```
int p=0; //total number of people on all flights
for(int j=0; j<n; j++)
	p += int(flights[j]); //add up number of people on each flights

float max10 = float(p)/6.0; //maximum number of people in each 10 minute interval

float interval[6]; //actual number of people in each interval
for(int k=0; k<6; k++)
	interval[k] = max10; //put max10 in each component of array interval[]

//cout << "n: " << n << "\np: " << p << "\nmax10: " << max10 << '\n';

int numFlights[6]; //number of flights in each interval
for(int c=0; c<6; c++)
	numFlights[c] = 0;

double timeInt0[n]; //flights in each time interval...
double timeInt1[n];
double timeInt2[n];
double timeInt3[n];
double timeInt4[n];
double timeInt5[n];

double value;
for(int a=0; a<n; a++)
{
	value = maxVal(flights,n);  //gets the maximum value in flights[]
	switch( maxTimeInterval(interval) ) //determines which time interval to put
	                                     //value in, by finding out the maximum
	                                     //value of seats left in any interval
	{
		case(0):
			numFlights[0]++; //increase number of flights in interval 0
			interval[0] = interval[0] - value;
			timeInt0[ numFlights[0] - 1 ] = value;
			break;
		case(1):
			numFlights[1]++; //increase number of flights in interval 1
			interval[1] = interval[1] - value;
			timeInt1[ numFlights[1] - 1] = value;
			break;
		case(2):
			numFlights[2]++; //increase number of flights in interval 2
			interval[2] = interval[2] - value;
			timeInt2[ numFlights[2] - 1] = value;
			break;
		case(3):
			numFlights[3]++; //increase number of flights in interval 3
			interval[3] = interval[3] - value;
			timeInt3[ numFlights[3] - 1] = value;
			break;
		case(4):
```

```
                numFlights[4]++; //increase number of flights in interval 4
                interval[4] = interval[4] - value;
                timeInt4[ numFlights[4] - 1] = value;
                break;
            case(5): //increase number of flights in interval 5
                numFlights[5]++;
                interval[5] = interval[5] - value;
                timeInt5[ numFlights[5] - 1] = value;
                break;
            default:
                cout << "something is wrong with the switch statement\n";
        }
    }

//outputs the distributed intervals onto the screen
    double tempSum = 0;
    cout << "Time Interval 0, :00->:09...\n";
    for(int b=0; b<numFlights[0]; b++)
    {
        tempSum += timeInt0[b];
        cout << timeInt0[b] << '\n';
    }
    cout << "Total # of flights in Time Interval 0 is: " << numFlights[0] << "\n";
    cout << "Total # of Average numbers of seats filled for Time Interval 0 is: "
        << tempSum << "\n\n";

    tempSum = 0;
    cout << "\nTime Interval 1, :10->:19...\n";
    for(int b=0; b<numFlights[1]; b++)
    {
        tempSum += timeInt1[b];
        cout << timeInt1[b] << '\n';
    }
    cout << "Total # of flights in Time Interval 1 is: " << numFlights[1] << "\n";
    cout << "Total # of Average numbers of seats filled for Time Interval 1 is: "
        << tempSum << "\n\n";

    tempSum = 0;
    cout << "\nTime Interval 2, :20->:29...\n";
    for(int b=0; b<numFlights[2]; b++)
    {
        tempSum += timeInt2[b];
        cout << timeInt2[b] << '\n';
    }
    cout << "Total # of flights in Time Interval 2 is: " << numFlights[2] << "\n";
    cout << "Total # of Average numbers of seats filled for time Interval 2 is: "
        << tempSum << "\n\n";

    tempSum = 0;
    cout << "\nTime Interval 3, :30->:39...\n";
    for(int b=0; b<numFlights[3]; b++)
    {
```

```
        tempSum += timeInt3[b];
        cout << timeInt3[b] << '\n';
    }
    cout << "Total # of flights in Time Interval 3 is: " << numFlights[3] << "\n";
    cout << "Total # of Average numbers of seats filled for Time Interval 3 is: "
        << tempSum << "\n\n";

    tempSum = 0;
    cout << "\nTime Interval 4, :40->:49...\n";
    for(int b=0; b<numFlights[4]; b++)
    {
        tempSum += timeInt4[b];
        cout << timeInt4[b] << '\n';
    }
    cout << "Total # of flights in Time Interval 4 is: " << numFlights[4] << "\n";
    cout << "Total # of Average numbers of seats filled for Time Interval 4 is: "
        << tempSum << "\n\n";

    tempSum = 0;
    cout << "\nTime Interval 5, :50->:59...\n";
    for(int b=0; b<numFlights[5]; b++)
    {
        tempSum += timeInt5[b];
        cout << timeInt5[b] << '\n';
    }
    cout << "Total # of flights in Time Interval 5 is: " << numFlights[5] << "\n";
    cout << "Total # of Average numbers of seats filled for Time Interval 5 is: "
        << tempSum << "\n\n";

    return 0;
}//end of main()

int maxTimeInterval(float interval[]) //finds the maximum number in interval[] and returns the index
{
    float value = -10000;
    int index;
    for(int i=0; i<6; i++)
    {
        if(interval[i] > value)
        {
            value = interval[i];
            index = i;
        }
    }
    return index;
}
double maxVal(double array[], int n) //returns the maximum value in given array
{
    double value = 0;
    int index;

    for(int z=0; z<n; z++)
```

```
        if(array[z] > value)
        {
            value = array[z];
            index = z;
        }

    array[index] = 0;
    return value;
}

void convert(double array[], int n) //converts flights from having number of seats available to average
number of passengers on flight
{
    for(int d=0; d<n; d++)
    {
        if(array[d] >= 34 && array[d] <= 85) //flights with seats from 34->85
            array[d] *= 0.8679; //average percentage of seats filled for this section
        else if(array[d] >= 128 && array[d] <=215) //flights with seats from 128->215
            array[d] *= 0.8194; //average percentage of seats filled for this section
        else if(array[d] = 350) //flights with 350 seats
            array[d] *= 0.7705; //average percentage of seats filled for this section
        else
            cout << "sorry, we do not have data for such a flight. thanks, y'all come”
                 << “ back now, ya hear?\n";
    }
    return;
}
```

SAMPLE OUTPUT for program

Sample Data: Airport A during Peak Hour. If number of flights entered is 46, and flights are entered such as appear in the Technical Information Sheet (TIS) of the problem description, then the following is the output:

```
Enter total number of flights to be distributed: 46
Enter the number of people on each of the 46 flights:
[flight data on Airport A during peak hour entered here…]
...thank you, please drive through to the next window
Time Interval 0, :00->:09...
269.675
116.355
116.355
116.355
73.7715
29.5086
29.5086
Total # of flights in Time Interval 0 is: 7
Total # of Average numbers of seats filled for Time Interval 0 is: 751.528


Time Interval 1, :10->:19...
176.171
116.355
```

116.355
116.355
104.883
39.9234
29.5086
29.5086
Total # of flights in Time Interval 1 is: 8
Total # of Average numbers of seats filled for Time Interval 1 is: 729.059

Time Interval 2, :20->:29...
158.964
158.964
116.355
116.355
104.883
39.9234
29.5086
29.5086
Total # of flights in Time Interval 2 is: 8
Total # of Average numbers of seats filled for time Interval 2 is: 754.461

Time Interval 3, :30->:39...
158.964
116.355
116.355
116.355
116.355
73.7715
29.5086
Total # of flights in Time Interval 3 is: 7
Total # of Average numbers of seats filled for Time Interval 3 is: 727.663

Time Interval 4, :40->:49...
158.964
116.355
116.355
116.355
116.355
39.9234
39.9234
29.5086
Total # of flights in Time Interval 4 is: 8
Total # of Average numbers of seats filled for Time Interval 4 is: 733.738

Time Interval 5, :50->:59...
158.964
116.355
116.355

116.355
104.883
73.7715
29.5086
29.5086
Total # of flights in Time Interval 5 is: 8
Total # of Average numbers of seats filled for Time Interval 5 is: 745.7

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